The Real Physics of Machine Vibration

by

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Abstract

Classical physics works well for static structures, rigid bodies, and zero frequency motion. It is even satisfactory for design of low speed machines. Low speed is defined as well below the first natural frequency bending mode of any component. When a structure becomes active, i.e. resonant, then response motion is non-linear and out-of-phase with the force input. The active component can function as a mechanical amplifier or filter. The consequences of this are unpredictable behavior, catastrophic failures, or wonderful new inventions. This paper will explore the dynamic environment that rotating machines encounter and how to deal with it.

Introduction

This is the age of rotating machines, specifically, high speed rotating machines that spin above several hundred rpm. All rotating machines generate some oscillation because perfect geometry and perfect balance are not possible. Rotating machines in the form of cart wheels have been around for at least 6,000 years, but they were not high speed. It is not known what form of dynamic analysis was carried out in their design, if any. More than likely it was an art with abundant trial and error. This was long before the physics of motion, linear or rotating, was codified in mathematics, or even correctly understood. The end of life for these prehistoric wheels was likely defined by abrasive wear or fracture from excessive strain. With the invention of iron, corrosion also became a failure mode.

Today, many machines never see wear and corrosion as their final exit because they fail from vibration induced forces. This oscillation causes joint softening, amplified motion that creates fluid leaks, electronic malfunctions, and fatigue cracking. Low speed machines might have an infinite life if we could solve the wear and corrosion failure modes, but would we even want obsolete machine designs to operate forever? Machine failures could then be a welcome blessing, pressing us to rethink something better. Better could be smaller, lighter, faster, cheaper, more efficient use of energy, or just a change in the process. The advent of high speed machines has forced us into more analysis in design, vibration monitoring, corrective methods of balancing, alignment, and resonance fixes. It is time to re-address the physics.

Vibration[1] ≡ “A periodic motion of the particles of an elastic body or medium in alternately opposite directions from the position of equilibrium when that equilibrium has been disturbed.”

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1. Merriam Webster's Collegiate Dictionary, Tenth Edition

This definition recognizes two real world machine conditions. One is that materials have some elasticity, and second, that a source of energy is needed to sustain vibration.

Most machines are connected some way to the Earth via a support structure. Machines are not islands unto themselves, but are coupled, both physically and dynamically to something else – be it a concrete foundation, a skid, a deck plate, or just plain dirt. The characteristics of that support structure determine, to a large extent, it's dynamic behavior via the natural frequency equation of a single degree of freedom system –

$\omega_n = \sqrt{k/m}$

ω_n = natural frequency, radians/second
k = stiffness
m = mass

The stiffness factor is largely determined by what is underneath the machine or what it is connected to. Bear in mind that the end user typically supplies the support structure. This means that the machine supplier cannot be held totally responsible for it's dynamic behavior at it's permanently installed location.

What are the consequences for machines with no solid foundation to the Earth, like aerospace vehicles, or even boats on water? First, and foremost, is that the vibration energy generated on-board remains on-board. With no good path to the Earth, then the vibration energy can only be dissipated within the damping present in the materials or joints of the machine itself. Boats have the advantage of the damping in the heavy water. Aircraft have much less fluid damping available in the relative wind. Space vehicles are on their own. However, space vehicles cannot be excited by the random fluid motion since there is none in a vacuum. The vacuum of space also complicates wear because fluid lubricants evaporate away faster in a vacuum. A good design principle is that all machines operate better the closer they are to concrete. For large and heavy machines, this principle is paramount to successful performance as long as the uneven concrete surface does not distort it's frame.

The second consequence of machines with no fixed supports to an infinitely stiff foundation is that the internal forces generated during machine operation will allow the frame to flex. One part of the machine structure can react with, and exchange energy with another part, and they can be out of phase with each other. This will affect internal alignments, causing operational problems and excessive wear. Newton's third law of action-reaction does not strictly apply because there will be time delays along the transmission path, the path can modify the amplitude, and the force can be out of phase with the resulting motion.

Third, with no solid foundation, spring isolators will not function properly. The assumption with isolators is that one side is immovable. For coil spring isolators under a machine on terra firma, the assumption is that what is under the spring is infinitely stiff. If the foundation is not sufficiently stiff, then force and motion will register under the spring and possibly exchange energy with the foundation. The resulting behavior depends on the relation between the natural frequency of the machine on it's springs and the natural frequency of the foundation.

A note on college vibration courses. The differential equations in engineering or physics classes are good to discuss concepts, but are not very useful for solving real world machine, or even structural, vibrations. They rely on unrealistic assumptions like:

1. Rigid bodies with no elasticity
2. Ideal springs with no mass
3. Point masses rather than distributed mass
4. Frictionless bearings
5. Fixed supports

The analytical solutions appear elegant from a mathematical perspective. Pythagoras and Plato would be proud and satisfied, but they do not account for the variables of people, less than ideal assumed conditions, unknown contingencies, and Murphy. The useful tools for addressing vibrations on machines and structures are:

1. Measurement and analysis
2. Balancing
3. Alignment
4. Resonance fixes
5. Identify the bad parts

Measurement and analysis involves using a spectrum analyzer to identify the frequencies and amplitudes of concern. The frequency is characteristic of the source. The vibration energy travels through the structure as a longitudinal compression wave from the source. It comes out unchanged, except for a slight decrease in frequency when there is significant damping. It is for this one simple physical phenomena that we are even able to do vibration analysis at all. It is difficult to imagine vibration analysis as a viable technology if the frequencies shifted as the wave moved through materials. The frequency will establish a footprint of the offending part. Vibration is channeled along the structural parts like a train confined to the rails. The amplitude is characteristic of the path. The path can filter, amplify, or attenuate the vibratory energy. The amplitude is the parameter that we use to judge the severity of the vibration. This is where analysis is used to interpret the squiggly lines. The analyst uses the frequency, the amplitude, where it is measured, the load on the machine, some visualization of the inner workings, and judgment to determine if the vibration is benign or serious.

Balancing and alignment are common defects on rotating machines. There are good instruments and procedures to do these corrective methods in the field. Alignment is a safe procedure done with the machine stopped, but balancing is a dangerous business. The machine must be started and stopped a number of times to measure it's original condition, again with a test weight on-board, and perhaps many more times to do trim balancing and verify good results. Each time, the proper coordination must be done correctly with configuring controls. The machine is stressed, both electrically and mechanically with each start. If there are latent defects in the machine or it's controls, then there is an opportunity to precipitate a failure. Test weights can fly off. The balancer must remember to de-energize the machine every time hands are placed on the rotor. Excessively large test weights can damage the rotor or nearby observers. The machine may be operated in an abnormal posture for balancing purposes, like maximum speed, dampers or valves closed, controls bypassed. Mass balancing is not for the faint of heart.

Concerning resonance, there are five known fixes.

1. Change speed
2. Change the natural frequency of the vibrating part
3. Add damping
4. Reduce the source energy with finer balancing or alignment
5. Dynamic absorber

Which specific fix to apply is the domain of advanced analysis which involves additional testing and some practical consideration of what is possible on that machine within the time constraints.

Identifying the "bad" or worn part is useful to plan repair in an efficient manner. This is the primary reason for establishing a predictive vibration monitoring program.

As a last resort, when these methods are exhausted and the problem is not fully understood, then stiffening and mass loading have the potential to reduce any measured vibration at a specific location. Be cautious though, as these last resort methods tend to increase local contact stresses at the bearings leading to accelerated wear.

Classical Physics

Newtonian, or classical, physics is based on particle motion, point masses, and rigid bodies. We know this not to be true in the real world, yet we continue to accept it. These are ideal, not real, conditions. The real world is distributed masses, elastic bodies, and relative motion within bodies with parts out-of-phase, in addition to motion of the body as a whole.

The point philosophy originated with the Greek philosophers Leucippus and Democritus. The point particle has no physical dimension and cannot have any elastic properties. Newton extended this ideal concept to develop wonderful laws of motion and gravitation. This corpuscular philosophy was extended to fluids, heat phenomena, and even light. It ignored waves.

Two fundamental laws of classical physics are Hooke's law, F = kx, and Newton's 2nd law, F = ma.

Hooke's law, in it's modern form relates stress to strain via the modulus of elasticity, but Robert Hooke initially postulated it in 1676 as F = kx. The stiffness constant, k, is composed of the cross section area, the length, and the modulus. These together have the same units as the force divided by the deflection.

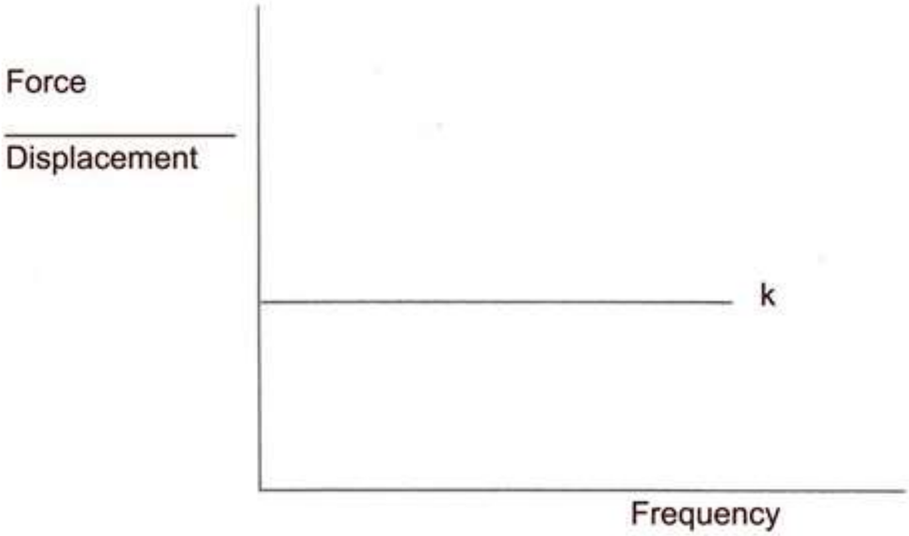

Fig 1. Hooke's law F/x = k, a constant. An ideal theoretical situation.

Newton never stated the second law, F = ma, directly in it's mathematical form. The concepts of force and motion that he described in the Principia were debated among philosophers for decades. Euler formally presented the mathematical formula F = ma approximately 100 years later in 1750.

In these formulas – F = force, newtons
k = stiffness, newtons per meter (specifically, static stiffness)
x = displacement, meters
m = mass, kilograms
a = acceleration, meters per second squared

Force is not measurable, even though it is displayed on a spring scale or an electronic readout from a load cell. Motion is measurable, and from the measured motion and a calibrated relationship between motion and force via the static spring constant, force is calculated and displayed. Force is an imaginary concept. In statics, a force may be defined as *any action* that tends to maintain or alter the position of a body, "or to distort it". Physically, point forces are impossible. The contacting surfaces flatten resulting in a small area. The high spots crush down to an area just large enough to support the load. This force spread over an area is really a pressure. This also produces an internal stress that is distributed. Hence, the body distorts.

The word "action" has an interesting history in physics that predates force. It has been debated by philosophers since Aristotle. Today, action is defined as energy multiplied by time, or conversely, energy divided by frequency. When frequency is considered, then action and force become wave phenomena.

These two equations give us an operational definition of force. Strictly by scientific philosophy, a quantity that cannot be measured does not exist. But this imaginary force has proven useful in engineering circles. Force times motion equals energy. Energy is not a physical "thing", but a process, and it is measurable.

Normally, in machine design, stiffness, k, is considered to be constant. To assume otherwise requires some experimentation to determine it's variability with temperature, or some other parameter. This is too much effort, so it is convenient to just assume it to not vary. But it does vary with frequency, rather substantially. If stiffness were truly constant, then the plot of force divided by displacement would be a constant number at all frequencies, fig 1.The frequency would not matter, and by the way, spring isolators would not work. The theory of isolation is based on the resonance curve of a single-degree-of-freedom system. For those of us in the profession of measuring vibration, we know that displacement changes around the resonant frequency of a single-degree-of-freedom system. Displacement grows larger, resulting in a smaller ratio of force divided by displacement, or a lower stiffness, figure 2.

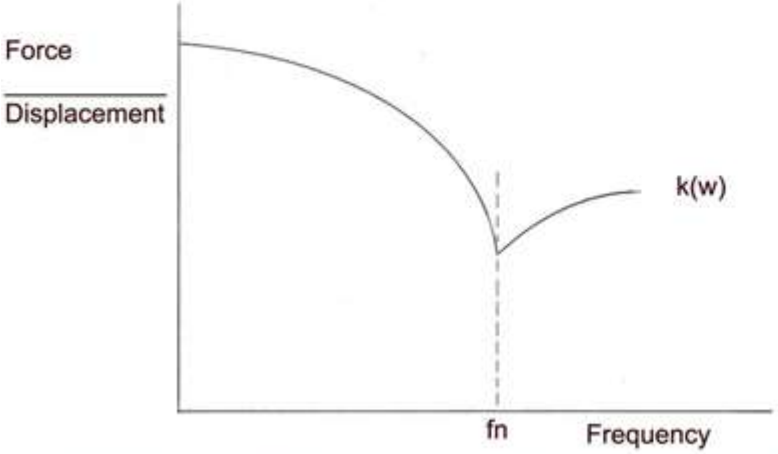

Fig 2. Dynamic stiffness decreases at the natural frequency, fn, in the real world and never fully recovers.

So stiffness is a function of frequency. This is called “dynamic stiffness”, or it's reciprocal, “compliance”. On machines, there will be multiple resonances, and consequently, multiple dips in the dynamic stiffness plot.

Newton's 2nd law, $F = ma$, suffers a similar fate. It defines acceleration as the ratio of force divided by mass, $a = F/m$. The acceleration would have nothing to do with frequency, so it would plot as a flat line, figure 3.

This assumes mass to be a constant, which is a good approximation when motion is much less than the speed of light.

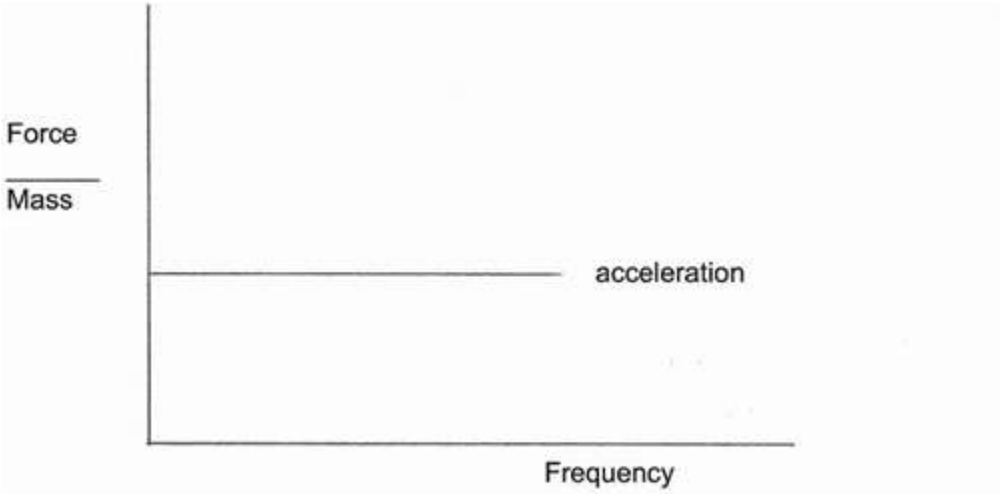

Fig 3. Newton's 2nd law with acceleration as a ratio of force divided by mass. Acceleration is a constant at all frequencies.

In a dynamic world, when oscillation becomes active, the constant mass resists a change in motion direction, exhibiting greater inertia. Masses do not “like" to shake at higher frequencies. Anyone who has measured acceleration during a run up or coast down of a machine has observed the acceleration motion to change with speed. The acceleration is not constant during speed changes. The acceleration increases with speed due to centrifugal forces of a residual unbalance, and to the dynamic material properties which are the main subject of this paper. The force increases as a quadratic curve in accordance with the centrifugal force equation –

$F = mr\omega^2$ F = force, newtons
m = mass, kilograms
r = radius of unbalance mass, meters
ω = frequency, radians per second

In addition, near a natural frequency, the acceleration plot versus frequency displays an amplification followed by a reduction. The combination of centrifugal force from a rotating machine and a resonance may look like figure 4.

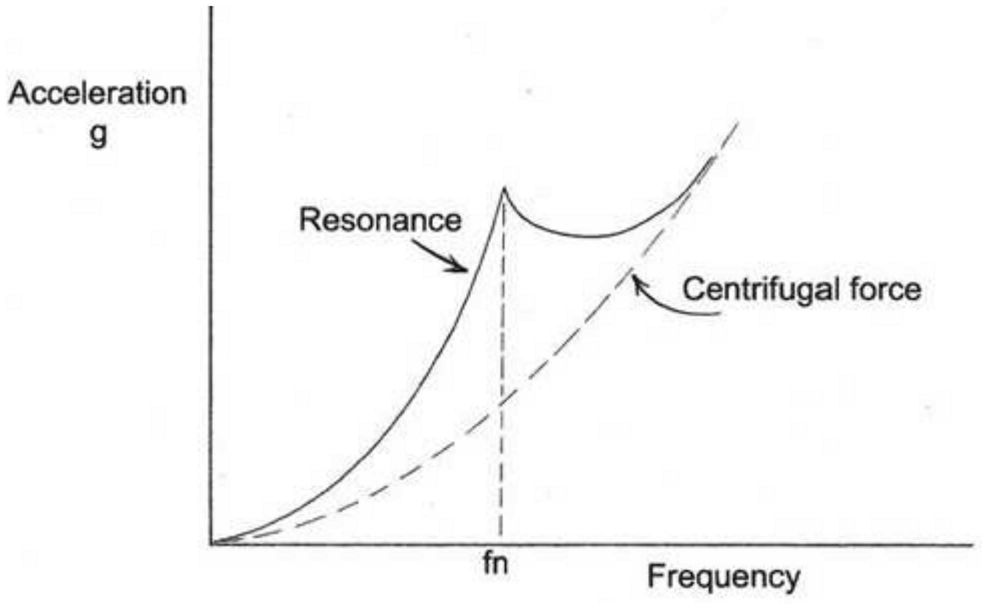

Fig 4. Acceleration versus frequency for any real rotating machine.

Plotting a resonance curve in acceleration is proof that Newton's 2nd law is not valid at all frequencies. To be fair, it is only valid at zero frequency. To preserve the linearity of Newton's 2nd law, we define an apparent mass, or a dynamic mass, that changes with frequency.

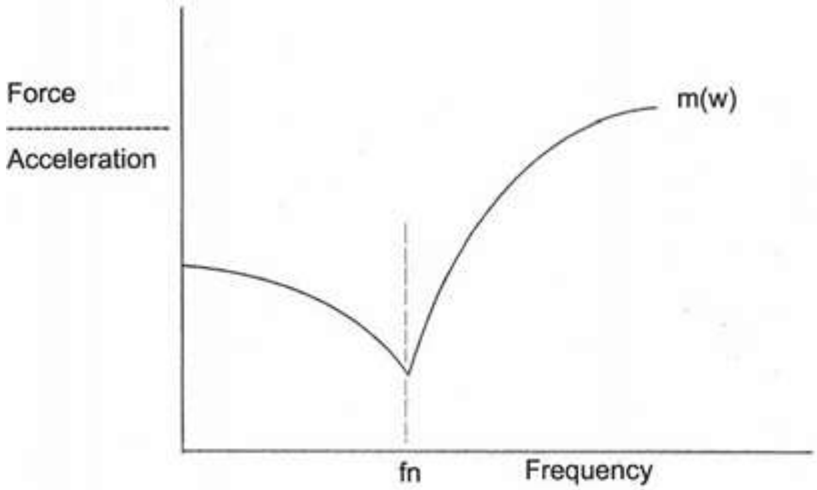

Fig 5. Apparent mass or dynamic mass decreases at the natural frequency, fn, then goes wild.

The force divided by acceleration ratio versus frequency will look like figure 5.

The reciprocal of dynamic mass is accelerance, and is also a function of frequency. Accelerance = 1/m = a/F. Real machine systems will have multiple dips in this dynamic mass curve. Mass, then, is not constant, at least conceptually. How much stranger can we imagine the world to be?

Formal physics and mechanical engineering curriculum teach mass to be a fundamental, and constant, quantity. Obviously, mass cannot be constant in all situations and variable with frequency at some times without some qualification. The qualification is that Newton's 2nd law is only strictly valid at zero frequency. Beyond that point, the world begins to look like “Jello”. I suppose, given enough confusing data, combined with a vivid imagination, we could wrap ourselves around a tree tightly enough to become one with the tree.

If motion was measured in velocity units, then similar definitions of mobility and mechanical impedance have been defined to keep the world linear. Linearity, like symmetry, are false perceptions of reality. These two perceptions, symmetry and linearity, besides making the math easier, have a certain zeal for the human psyche.

So amplitude alone, as a judgment criteria for vibration severity comes up short. Frequency must be a factor for consideration. These two fundamental principles of physics, Hooke's law $F= kx$ and Newton's 2nd law $F = ma$, work well for designing static structures. In a machine environment, with oscillatory motion, they are incomplete. Mechanical wear and fatigue failures are better characterized with frequency in addition to amplitude. This is the definition of a wave phenomena. Newtonian physics ignores waves.

Newton's 3rd law of action-reaction also needs qualification. It only applies at the point of contact, and not some distance away from that point because vibration travels through materials at the speed of sound in that material. There can be some time delay. An impact will generate a stress wave through the material which will have a frequency. There will also be a phase delay, implying a change in direction. There will be some deformation with heat generation and sound energy emission. No materials are perfectly elastic and there are no rigid bodies. Newton's 3rd law applies approximately for static structures, rigid bodies, no springiness, and zero frequency. Rotating machines do not satisfy those conditions.

The Proposed Solution

For most of machine design history, machines were built, then tested. The weaknesses were addressed, then modified and retested. That is still a valid engineering approach. In fact, buyers of machines should not trust any new design that has not been tested on a similar foundation. The testing should be dynamic, with vibration measurements at each bearing, in all three orthogonal directions, throughout the operating speed range, and under load. This is akin to test driving a car before purchase. If not witnessed at the factory, then a test report from the original equipment manufacturer of the machine operated on their test stand prior to shipment is a suitable substitute. The foundation prepared on site is a variable that the final customer is responsible for. Acceptance testing on site with another vibration survey certifies that the machine/support structure is suitable for service.

The trend in the manufacturing environment is to reduce cost by economizing on materials (thinner and lighter), reduce or even eliminate testing at the factory, and to shorten development time by deleting the prototype cycle of build and test. The development of new machine designs then relies on analytical modeling. This management trend to reduce factory cost has been an economic boon for vibration analysts who get to finish the design on site.

Analytical modeling demands some assumptions about material properties and how forces transmit through the elements. The assumptions depend on linearity, superposition, symmetry, and Newton's 3rd law of action-reaction. These are very big assumptions in a world of non-linear material properties and components that are flexible, elastic, plastic if overstressed, and that possess natural frequencies. The bearings are not totally fixed. Friction is only a guesstimate. The balance condition is assumed. Alignment on-site is unknown. The installed conditions mating to the foundation can introduce strain, which changes stiffness properties. Nature itself can be chaotic with temperature, wind loading, and random energy inputs from combustion and other nearby machines. Much of this is unforeseen and cannot be modeled. Hence, the need for on-site testing. But the analytical model should at least produce a design that passes muster at the factory. To that end it needs to be adjusted to account for frequency.

The first adjustment is to abandon the linear assumptions that only apply at zero frequency. The final form of Hooke's law and Newton's 2nd law will take into account frequency. The force input from rotational residual unbalance is a function of time, therefore, the responding motion is also a function of time. This can be transposed into functions of frequency.

$$F(\omega) = kx(\omega) \qquad F(\omega) = ma(\omega)$$

The end result could be similar to AC circuit formulas that replace resistance, R in Ohm's law $V = IR$, with impedance $V = IZ$. The impedance is a function of frequency based on the inductance and capacitance in the circuit. Inductance in AC circuits is analogous to mass in mechanics. Capacitance in AC circuits is analogous to the reciprocal of stiffness in mechanics.

$$L \leftrightarrow \text{mass} \qquad C \leftrightarrow 1/k$$

The combination of the two quantities, mass and stiffness, which are functions of frequency (in a conceptual sense) will determine the natural frequencies. Since forces travel through materials at the speed of sound in that material as a stress wave, then phase relationships need to be considered as the wave arrives at the different points with some timing delay. This implies that force is a wave and has characteristics of frequency in addition to amplitude. Indeed, if we consider mass to be constant, then force must be a function of frequency because acceleration is variable with frequency. Force, in a dynamic world, is actually a wave phenomena.

$$F(\omega) = ma(\omega)$$

The linear formulas used for so long in engineering design are a starting point, but are not finished. If dynamic testing, with adjustments and retesting are to be deleted in machine development, then frequency is a necessary addition to the analytical modeling. This is being done in finite element modeling, but is not refined enough to be accurate with confidence. There are too many uncertainties in assumptions.

A Brief History of Mathematics

The Egyptians used number "know-how" to build accurate pyramids and other large structures. This was arithmetic. With arithmetic, we can capture some characteristic numbers like peak amplitude, RMS (root mean square), and frequency, but the live motion is absent. We can also put a number on damping and energy. A video shows everything in real time, but the motion seen is too small to make sense unless it is amplified. We can show an amplified video today, but it is hard to put numbers on it.

Geometry is the mathematics developed by the Greeks, who inherited the number know-how for building monuments from the Egyptians. It is basically arithmetic extended to two and three dimensions for characterizing static structures. The Greeks extended the arithmetic into general axioms and postulates, and subsequently built a philosophy of heavenly body motions with their perfect forms, which, unfortunately, was incorrect for about 2,000 years. There is no way to capture the resonant behavior of machines and structures with geometry. Geometry is static only.

The Arabs developed algebra to further generalize natural phenomena into equations that, supposedly, could be applied to any form of motion as long as the unknown variables could be identified. Algebra can describe a continuous curve of two variables, one independent and the other dependent. The equations were linear, or first order. Subsequently, second and even third order general equations were solved by European mathematicians. Machine dynamic behavior is beyond the ability of even 4th order algebra to describe. Algebra is not the correct form of mathematics to describe machine motion, primarily because it does not consider frequency, nor accommodate non-linear behavior well.

Calculus is the mathematics of motion in one direction, not oscillation. Differential calculus captures the rate, while integral calculus captures the total excursion after some time. The integral will be zero for oscillation. Historically, the fundamental unit of machine vibration has been the maximum amplitude in displacement or velocity because these were easy to measure. That amplitude has been reported as peak or RMS, along with the frequency. However, that is only a snapshot of the dynamic motion and assumes a sinusoidal waveform. It does not capture the true shape of the waveform, nor the force, nor the energy.

The present method for dealing with dynamic machine motion is graphical i.e. the resonance curve. That curve takes into account the ratio of machine generated motion (or forced vibration) and the support structure response (natural frequency). This works for one variable, the speed. A new form of general mathematics is needed that is based on the natural frequency as the fundamental unit, but also incorporates the energy, caused by periodic unbalance and random inputs, along with damping. The damping can be viscous, frictional, or mass, and is characterized by the two-channel forced/response function. Eventually, this proposed mathematics will need to be extended to multiple degrees of freedom and cross coupling. To summarize, the factors that influence machine motion are --

a. Input energy (unbalance or random energy)
b. Mass
c. Stiffness
d. Damping
e. Frequency Ratio

The Consequences

Our acceptance of machines is based on how much or how little they shake. We have on board sensors for detecting shake through our skin. The shaking is actually wasted energy leaking from the machine. Energy flows through a machine as part of it's process of doing work and some of this energy leaks out as vibration that we sense to judge it's condition. These measurements are real, but not the complete picture because all measurement systems filter. The filtering is based on frequency. For example, the pressure sensors in our skin are good at sensing and judging motion below a few hundred hertz. At higher frequencies they become numb. Our ears are sensitive to these higher frequencies, but begin losing it below 50 hertz.

The real physical world is hidden behind a veil of our senses, which can only perceive a portion of reality, filtered by frequency. We can never directly perceive the whole of what surrounds us, only part of it. To a deaf person with no feeling, every machine would be O.K. The consequences of zero measurement is 100% acceptance. Similarly, if broadband frequency is absent from analysis or modeling, then we have placed earmuffs on what we can hear and what we allow ourselves to perceive.

Without frequency, we cannot detect modes of shaking. Vibrating modes are a consequence of symmetry. When we produce machine parts with uniform cross sections, homogeneous and isotropic materials, and symmetrically places support points (like for hold down bolts), we are inviting resonant mode shapes in. Symmetry is a bad design practice for structures and machines because it supports resonant modes. A four bladed propeller fan rotating near four support beams (like cooling towers), is one bad example. Flat plates with uniform support locations are the worst. Building floors supported by evenly spaced columns are inviting drumming modes. Machine support plates with symmetrically placed hold down bolts are likely to resonate at specific speeds. Castings are a good example of non-symmetrical shapes with varying thickness and non-repeating geometry. They just do not “drum”.

Force must still be considered as an imaginary concept since we cannot measure it directly. How do we imagine it? We can imagine force as a vector, as a push or pull, or as something that makes a body change it's state of motion. In a dynamic machine environment with oscillation, force has an amplitude and a frequency. It travels through materials as a wave. It can have a time delay from the point of application to the measured motion location. The force can be amplified or filtered like any other wave phenomena. The energy of the wave reacts with matter and can be transmitted, absorbed, or reflected. The resulting behavior is frequency dependent. In this context, force is a wave phenomena.