Analysis of Laminated Composites Subjected to Impact

Sergey Ugrimov(✉), Natalia Smetankina, Oleg Kravchenko, and Vladimir Yareshchenko

A. Pidgorny Institute of Mechanical Engineering Problems of the National Academy of Sciences of Ukraine, 2/10 Pozharskogo Street, Kharkiv 61046, Ukraine
sugrimov@ipmach.kharkov.ua

Abstract. The paper proposes both theoretical and experimental approaches to the analysis of laminated composite response to impact loading. For theoretical modelling of dynamic behavior of a composite, the generalized model is used that takes into account the spatial character of deformation on near to the impact point. This model is based on a power series expansion of the displacement vector component in each layer for the transverse coordinate. The results of calculations are compared with the data obtained by other researchers for the case of low-velocity impact, as well as with the experimental data obtained by ourselves at medium-velocity impacts on composite panels. In the experimental study, maximum deflections of composite samples during the impact of an indenter were investigated. A pneumatic gun was used to launch the indenter, and a crusher was used to register the maximum deflections. An experimental study of the response of an eleven-layer fiber-glass composite to indenter impacts at different velocities was performed. For launching, the 600 g indenter was used. It is established that the calculation results and experimental data are in good agreement.

Keywords: Layered composite · Impact · Stress-strained state · Experiment

1 Introduction

Thin-walled composite shell structures are widely used in aerospace engineering, shipbuilding, chemical industry, and automobile industry [1, 2]. They work under conditions of both stationary and non-stationary force loads. Dynamic loads are observed in operating modes, and also in some emergency situations. Such loads occur during various explosions, when an aircraft collides with a bird, when an aircraft tire fragment hits the composite wing of the aircraft, when the composite cabin of an electric locomotive is hit by solid objects from the oncoming train, and so on [3–5]. These are practically important tasks that are considered in the design of composite structural elements in various fields of technology.

Particularly dangerous to thin layered structures is the transverse impact of a rigid projectile [6, 7]. In composites, such a localized intense load can lead to matrix destruction, fiber damage, and structure delamination. This is why special attention is

M. Nechyporuk et al. (Eds.): ICTM 2020, LNNS 188, pp. 234–246, 2021.
https://doi.org/10.1007/978-3-030-66717-7_19

paid to both theoretical and experimental methods of analyzing the stress-strained state of thin-walled composite structures under impact loading [8].

When solving the problem of impact on layered anisotropic structures, the known mathematical difficulties associated with an adequate description of the multilayer structure of composite [9–13] and the need to solve a dynamic problem [10–13], are aggravated by the presence of a localized load [10], which in general acts on a previously unknown area. This area must be determined in the process of solving the problem itself. In addition, the impact in the composite excites a wide range of oscillations with different frequencies, which imposes additional requirements for the structural model being applied.

All these problems result in a very limited number of works on the analysis of the response of laminated composite structures to the impact of elastic bodies [6, 8, 12, 13]. Typically, such problems are solved by numerical methods based on the discretization of a complex domain and its boundary, such as the finite element method (FEM). There are very few works devoted to analytical methods for solving such problems, especially for layered structures having a noncanonical shape. In this case, the immersion method [14], and the method of R-functions [15] can be used. With this, the behavior of a multilayer structure is usually described by low-order shell theories, and the process of impact interaction is described by Hertz's law or its modifications. Some problems of modeling the response of a composite to the low-velocity impact were considered in [6, 12, 13, 16]. In [6, 12, 13], refined first-order theories were used to model the behavior of a composite, which do not take into account the compression and nonlinear nature of stress distribution over the composite thickness. In [13], the solution was obtained by an analytical method, and in [6, 16], with the use of the FEM.

Nosier *et al.* [12] applied a refined discrete-structural theory of layered plates and investigated six models to describe the loading model. In the first five, the domain of interaction of the indenters and plates is proposed to be known. As a result, the problem is reduced to a nonlinear integral equation similar to Tymoshenko's equation. The impact force is interpolated by the Legendre and Hermite polynomials. In the sixth one, based on Hertz's law, the time dependence of the contact area was taken into account, which also led to the need to solve a nonlinear integral equation.

Pierson and Vaziri [13] obtained an analytical solution for the problem of analyzing the response of simply supported composite plates to the low-velocity impact. The equations of motion for the plates were based on the Whitney and Pagano first-order theory. Local indentation was taken into account on the basis of Hertz's law, and the coefficients that enter into this dependence were determined experimentally. The solution to the problem was based on the decomposition of displacements into Fourier series. The domain of integration was divided over time into equal segments, at each of which the impact force was assumed to be constant.

Tan and Sun [17] proposed a modified Hertz's law to improved description of impact on composite targets. The dependence was obtained experimentally in the study of the response of graphite-epoxy structures subjected to impact.

Choi and Hong [18] presented the results of theoretical and experimental studies of the response of layered composite plates to the impact. They applied both a high-order theory and the FEM. The impact force is described by modified Hertz's law. It is established that for a more accurate description of the behavior of composite plates at impact, it is necessary to apply a high-order theory.

In [10], it was proved that with localized impact loading, in the analysis of stresses in even thin composites, it is necessary to use high-order theories, and first-order theories can be used only for the approximate calculation of forces and displacements.

The presence of various assumptions in the models of layered composites, as well as simplifications in the description of interaction of indenter and composite structure requires, at the final stage of design of critical structures, conducting an experimental test of their response to impact. This requires using special equipment, namely launch devices, as well as equipment for registering deflections and strains. Low-velocity impact tests are usually performed by simply dropping the impactor onto a target [19, 20]. When studying the response of structures to medium- or high-speed impacts, it is already necessary to use special acceleration devices [4, 8, 20]. To register the behavior of structures during impact, high-speed video recording can be used, which allows one to assess the behavior of the entire structure, a variety of crushers to register the maximum structural displacement. One of the most common methods of registering strains is the method of dynamic wide-range strain gauging, which allows registering the time-dependent change in plane strains at a point [4]. But the rapidity of the deformation process requires the use of equipment with a high clock frequency. All this makes it difficult to perform the experiment.

The more detailed review of simulation and experimental study of composite structures subjected to low-velocity impact can be found, for example, in works Abrate [8], Patil *et al.* [19], Cantwell and Morton [20], Panettieri *et al.* [21].

This paper proposes a theoretical approach to the analysis of the response of a composite subjected to the impact loading. The approach is based on the hypotheses of the generalized theory of multilayer structures, which allows one to investigate laminated structures under localized loads.

2 Theoretical Modeling

2.1 Mathematical Model of Laminated Composite

A laminated composite consists of I layers of constant thickness, h_i is the thickness of the i-th layer (Fig. 1).

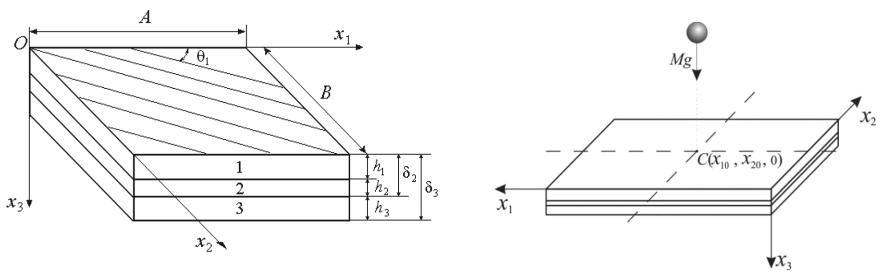

Fig. 1. Laminated composite. Impact problem.

The layers are made of orthotropic materials, and θ_i is the reinforcing angle in the i-th layer. The reinforcement directions in each layer are assumed to be parallel to the coordinate axes Ox_1, Ox_2. It is assumed that the contact between the layers excludes their delamination and mutual slipping.

The behavior of the layered plate is described by equations of the generalized theory of multilayer plates [10]. The displacement of a point on the i-th layer is described by the following kinematic relationships:

$$u_\alpha^i(x_1,\ x_2,\ x_3,t) = u_\alpha + \sum_{k=1}^{K_\alpha}\left[\sum_{j=1}^{i-1} h_j^k u_{\alpha k}^j + (x_3-\delta_{i-1})^k u_{\alpha k}^i\right], \qquad (1)$$

where $h_j^k = (h_j)^k$, $\delta_i = \sum_{j=1}^{i} h_j$, $\delta_{i-1} \le x_3 \le \delta_i$, $i = \overline{1,I}$; u_α^i $(\alpha = \overline{1,\ 3})$ are the displacements of a point in the i-th layer in the direction of the coordinate axes Ox_α; u_α, $u_{\alpha k}^i$ are the coefficients of expanding displacements into power series, which are functions of the arguments x_1, x_2, t; K_α are the maximum powers of the transverse coordinate for plane (α = 1, 2) and transverse (α = 3) displacements of the i-th layer. The parameters K_1 and K_2, which describe the number of retained power series terms for plane displacements, will be the same and equal to K, while the parameter K_3, which describes the number of retained power series terms for transverse displacements, shall be equal to L. The generalized theory shall be designated by the number of retained terms in power series (1) for plane and transverse displacements, viz., theory $\{K, L\}$.

By varying the number of retained terms in power series (1), it is possible to obtain two-dimensional approximations of the stress-strained state with different accuracy. Particular cases of the generalized model are Grigoliuk's model [22], the refined first-order theory, which takes into account the influence of the transverse normal and shear strains in each layers [23], as well as the high-order theory [24].

The strains in each layer are supposed to be small and are described by the linear relationships. With account of the accepted hypotheses (1), the the strain tensor of a point in the i-th layer ($\varepsilon_{\alpha\beta}^i$) take the form

$$\varepsilon_{\nu\nu}^i = u_{\nu,\nu} + \sum_{k=1}^{K}\left[\sum_{j=1}^{i-1} h_j^k u_{\nu k,\nu}^j + (x_3-\delta_{i-1})^k u_{\nu k,\nu}^i\right],$$

$$\varepsilon_{33}^i = \sum_{\ell=1}^{L} \ell(x_3-\delta_{i-1})^{\ell-1} u_{3\ell}^i,$$

$$\varepsilon_{12}^i = \frac{1}{2}\left(u_{1,2} + u_{2,1} + \sum_{k=1}^{K}\left[\sum_{j=1}^{i-1} h_j^k(u_{1k,2}^j + u_{2k,1}^j) + (x_3-\delta_{i-1})^k(u_{1k,2}^i + u_{2k,1}^i)\right]\right),$$

$$\varepsilon^i_{\nu 3} = \frac{1}{2}\left(\sum_{k=1}^{K} k\,(x_3 - \delta_{i-1})^{k-1} u^i_{\nu k} + u_{3,\nu} + \sum_{\ell=1}^{L}\left[\sum_{j=1}^{i-1} h^{\ell}_j u^j_{3\ell,\nu} + (x_3 - \delta_{i-1})^{\ell} u^i_{3\ell,\nu}\right]\right). \tag{2}$$

Applying hypotheses (1) yields a displacement field, which is continuous over the pack thickness, and ensures the continuity of the strains ε^i_{11}, ε^i_{22} and the piecewise continuity of the transverse strain ε^i_{33}, $\varepsilon^i_{\nu 3}$ (2) over the pack thickness. Therefore, within the theory being suggested, it is possible in principle to satisfy the conditions of contact between the layers with a specified accuracy.

The relation between the components of strain tensors and the stresses for the orthotropic case being considered has the form [10]

$$\begin{pmatrix} \varepsilon^i_{11} \\ \varepsilon^i_{22} \\ \varepsilon^i_{33} \end{pmatrix} = \begin{pmatrix} 1/E^i_1 & -\nu^i_{21}/E^i_2 & -\nu^i_{31}/E^i_3 \\ -\nu^i_{12}/E^i_1 & 1/E^i_2 & -\nu^i_{32}/E^i_3 \\ -\nu^i_{13}/E^i_1 & -\nu^i_{23}/E^i_2 & 1/E^i_3 \end{pmatrix} \cdot \begin{pmatrix} p^i_{11} \\ p^i_{22} \\ p^i_{33} \end{pmatrix}, \quad \begin{array}{l} \varepsilon^i_{12} = p^i_{12}/2G^i_{12} \\ \varepsilon^i_{13} = p^i_{13}/2G^i_{13}\ , \\ \varepsilon^i_{23} = p^i_{23}/2G^i_{23}. \end{array}$$

where $E^i_\alpha, \nu^i_{\alpha\beta}$ are Young's modulus and Poisson ratios, G^i_{12}, G^i_{13}, G^i_{23} are shear moduli, and $p^i_{\alpha\beta}$ is the stress tensor for the i-th layer.

Forces and moments in the i-th layer are determined by the formulas

$$N^{ik}_{\alpha\beta} = N^{ik}_{\beta\alpha} = \int_{\delta_{i-1}}^{\delta_i} (x_3 - \delta_{i-1})^k p^i_{\alpha\beta} \mathrm{d}x_3,\ \alpha, \beta = \overline{1,3},\ k = \overline{1,K}, \quad i = \overline{1,I}.$$

The equations of motion for the forces and moments have the form [10]

$$\sum_{i=1}^{I}\left[L^i_\alpha - I^i_{\alpha 1}\right] + q^1_\alpha = 0,$$

$$N^{i\,k_\alpha}_{1\alpha,1} + N^{i\,k_\alpha}_{\alpha 2,2} - k_\alpha N^{i\,k_\alpha - 1}_{\alpha 3} + h^{k_\alpha}_i \sum_{j=i}^{I-1}\left[L^{j+1}_\alpha - I^{j+1}_{\alpha 1}\right] - I^i_{\alpha k_\alpha + 1} = 0, \tag{3}$$

where $L^i_1 = N^{i0}_{11,1} + N^{i0}_{12,2}$, $L^i_2 = N^{i0}_{22,2} + N^{i0}_{12,1}$, $L^i_3 = N^{i0}_{13,1} + N^{i0}_{23,2}$;

$$I^i_{\alpha r} = \frac{\rho_i h^r_i}{r}\left(u_{\alpha 0,tt} + \sum_{k=1}^{K_\alpha}\left[\sum_{j=1}^{i-1} h^k_j u^j_{\alpha k,tt} + \frac{r h^k_i}{k+r} u^i_{\alpha k,tt}\right]\right),\ k_\alpha = \overline{1,K_\alpha},\ i = \overline{1,I},\ \alpha = \overline{1,3}.$$

The boundary conditions on the support contour for a simply supported rectangular plate are given below at $x_1 = 0$, $x_1 = A$,

$$\sum_{i=1}^{I} N_{11}^{i0} = 0, \quad u_2 = 0, \quad u_3 = 0, \quad N_{11}^{ik_1} + h_i^{k_1} \sum_{j=i}^{I-1} N_{11}^{j+10} = 0, \quad u_{2k_2}^{i} = 0, \quad u_{3k_3}^{i} = 0;$$

at $x_2 = 0$, $x_2 = B$,

$$u_1 = 0, \ \sum_{i=1}^{I} N_{22}^{i0} = 0, \ u_3 = 0, \ u_{1k_1}^{i} = 0, \ N_{22}^{ik_2} + h_i^{k_2} \sum_{j=i}^{I-1} N_{22}^{j+10} = 0, \ u_{3k_3}^{i} = 0. \quad (4)$$

Equations of motion (3) can be written in terms of displacements

$$\Omega \ \cdot \overline{U}_{,tt} - \Lambda \cdot \overline{U} = \overline{Q}, \quad (5)$$

where $\overline{U}$ is a vector whose components are the sought for functions $\overline{U}^{T} = \left(u_\alpha, \ u_{\alpha k_\alpha}^{i} \right)$, $\alpha = \overline{1,\ 3}$, $i = \overline{1, I}$, $k_\alpha = \overline{1,\ K_\alpha}$, Λ, Ω are the symmetrical matrixes of stiffness and mass of order $(2K + K_\alpha)I + 3$ [10]; $\overline{Q}$ is a vector whose components are a function of an external force applied to the external surface of the layered plate $\overline{Q}^{T} = (q_1,\ q_2,\ q_3,\ 0, \ldots,\ 0)$.

Equations of motion (5) and boundary conditions (4) are supplemented with zero initial conditions

$$u_\alpha = u_{\alpha k_\alpha}^{i} = 0, \quad u_{\alpha,t} = u_{\alpha k_\alpha, t}^{i} = 0, \quad \text{at}\, t = 0. \quad (6)$$

Hence, the dynamic behavior of a layered composite is described by the system of Eqs. (5), as well as boundary conditions (4) and initial conditions (6). The method of solving the obtained system of equations is based on the expansion of the sought functions u_α, $u_{\alpha k \alpha}^{i}$ $(\alpha = \overline{1,\ 3}, \quad k_\alpha = \overline{1,\ K_\alpha}, \qquad i = \overline{1, I})$ and the external load q_α into trigonometric series by functions $B_{\alpha mn}(x_1, x_2)$ satisfying the boundary conditions.

$$\left[u_\alpha,\ u_{\alpha k}^{i},\ q_\alpha\right] = \sum_{m=1}^{m1} \sum_{n=1}^{n1} \left[\Phi_{\alpha mn}(t),\ \Phi_{\alpha kmn}^{i}(t),\ q_{\alpha mn}(t)\right] B_{\alpha mn}(x_1, x_2),$$

where $m1$, $n1$ is the number of the terms retained in the series.

For a simply supported rectangular plate, function $B_{\alpha mn}(x_1, x_2)$ has the form

$$B_{1mn} = \cos c \sin d,\ B_{2mn} = \sin c \cos d,\ B_{3mn} = \sin c \sin d,\ c = m\,\pi\, x_1/A,\ d = n\,\pi\, x_2/B.$$

As a result, the problem on non-stationary deformation of a laminated composite for each pair of values m and n is reduced to integrating a system of ordinary second-order differential equations with constant coefficients and zero initial conditions.

2.2 Impact on a Laminated Composite

Let us investigate the process of non-stationary deformation of a horizontally located simply supported rectangular layered plate under low-velocity transverse impact (Fig. 1) [5, 10]. The impact is delivered in the middle of the outer surface of the first plate layer by a ball of radius R and mass M. At the moment of collision with the plate, the ball has a velocity V_0. The impact force and the area of contact are unknown in advance and must be determined in the process of solving the problem itself, so it is necessary to consider the mutual displacement of the plate and the indenter. The system of equations that describes the behavior of the plate is supplemented by the equations of motion for the indenter, as well as the condition of joint displacements of indenter and composite plate.

With this, it is convenient to specify the shape of the area and the nature of load distribution, and to determine their parameters while solving the problem. There are several classic models of the load area: a point load area, a rectangular area, and a circular area [12]. The most realistic picture can be obtained by using a circular area, especially for isotropic bodies. During impact on orthotropic plates, the load area has an elliptical character, but as a first approximation, a circular area is allowed to be used, which gives good results [12]. Therefore, we assume that the area of interaction between the indenter and the orthotropic plate is a circle of radius $r(t)$.

The character of load distribution over the contact area is unknown. To study it in the contact area, different mathematical models are used, for example, uniform and sinusoidal distributions [12]. However, a more accurate model is the ellipsoidal stress distribution over the contact area [12, 25]. In the study of impact, it will be used to model the distribution of stresses in the contact area.

Thus, the area of interaction between the indenter and plate is assumed to be a circle of radius $r(t)$, with the contact pressure being distributed over the contact area according to the law

$$q_3(x_1, x_2, t) = P_0(t)\left[1 - \frac{(x_1 - x_{10})^2 + (x_2 - x_{20})^2}{r^2}\right]^{0.5}, \tag{7}$$
$$q_1(x_1, x_2, t) = q_2(x_1, x_2, t) = 0,$$

where x_{10} and x_{20} are the coordinates of the ball and plate contact point at the initial instant of time.

The contact force with account of (7) is.

$$P(t) = \iint_S q_3 \mathrm{d}S = (2/3)P_0\pi a^2,$$

where S is the contact interaction area.

The equation of motion for centre of mass of the indenter and the initial conditions have the form [10]

$$Mz_{,tt} = Mg - P,\ z(0) = 0,\ z_{,t}(0) = V_0, \tag{8}$$

where $z = z(t)$ is the indenter displacement, P is the contact force, and g is the gravity factor.

The condition of the joint displacement of the indenter and composite plate is

$$u_3^1(x_{10},\ x_{20},\ 0,\ t) + a(t) - z(t) \geq 0, \tag{9}$$

where $a(t)$ is the contact approach of the ball and plate in the contact point.

The indenter and plate come into contact when inequality (13) becomes equality.

Contact approach is found using Hertz's law $a = kP^{2/3}$ [10, 12, 13, 26]. The coefficient k, which depends on materials and shapes of interacting bodies, for a contact of a ball with an isotropic plate has a rather simple expression obtained by Dinnik [25]. During impact on an orthotropic half-space, a similar expression for the coefficient k is absent [12]. In practice, the coefficient k is determined from an experiment or formulas for the isotropic case and averaged mechanical characteristics for the orthotropic half-space are used, for example $\nu_a = (\nu_{12} + \nu_{21})/2$, $E_a = (E_1 + E_2)/2$, where E_a, ν_a are the averaged values of Young's modulus and Poisson's ratio for the first layer [12].

The radius of the contact area $r(t)$ is computed using the formulas [10, 12]

$$r(t) = [(3/16) \cdot P(t)\ \cdot R \cdot (\theta + \theta_1)]^{1/3},\ \theta_1 = 4(1 - \nu_a^2)/E_a, \quad \theta = 4(1 - \nu^2)/E,$$

where E, ν are Young's modulus and the Poisson ratio for the ball material.

Thus, the non-stationary deformation problem of a laminated composite at impact is reduced to the integration of a system of equations describing the behavior of the composite plate (4)–(6), together with the equation of motion for the indenter (8) and the condition of the joint displacements (9). The method of solving the obtained system is described in detail in [10].

2.3 Numerical Results

The response of a symmetric ten-layer composite $(0°/90°/0°/90°/0°)_s$ to the impact of a steel ball that has a diameter of 12.7 mm, a mass of 8.5 g and an initial velocity of 3 m/s is considered [13]. The geometric parameters of the composite are $A = B = 0.2$ m, and $\delta_I = 2.69$ mm. The properties of the material are $E_{11} = 120$ GPa, $E_{22} = E_{33} = 7.9$ GPa, $G_{12} = G_{23} = G_{13} = 5.5$ GPa, $\nu_{12} = \nu_{13} = \nu_{23} = 0.3$, $\rho = 1580$ kg/m^3. The coefficient k that is used in Hertz's law was the same as in [10, 12, 13] for this problem.

Fig. 2 shows both the change in time for the contact force and deflections under the point of impact. The solid line shows the analytical solution to the two-dimensional problem [13], the dots show the solution by the FEM [12, 13], and the dotted line shows the solution according to the generalized model {7, 6}. The figure shows that in the time interval being investigated, the rebound is followed by a re-collision, and the results of calculations according to the proposed theory are in good agreement with the known solutions.

In the calculation, the generalized model {7, 6} was used, but for the general analysis of displacements and the contact force, lower-order models can be used [10]. High-order theories are necessary for a detailed analysis of stresses and strains in composite layers [10].

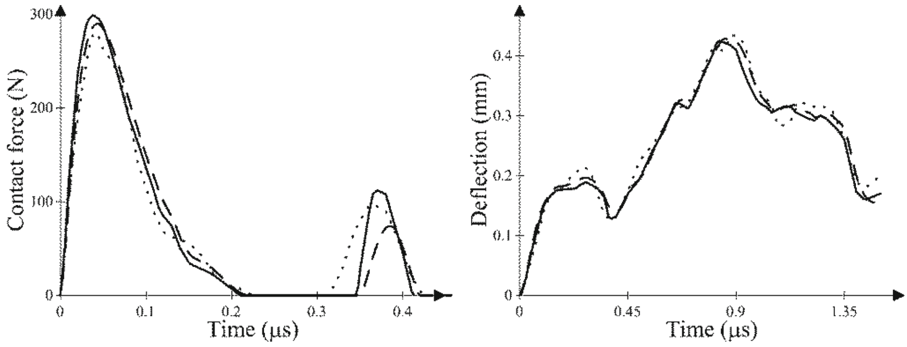

Fig. 2. Change of the contact force and deflections in time.

3 Experimental Setup and Results

For the experimental research, the test stand of the A. Pidgorny Institute of Mechanical Engineering Problems of the NAS of Ukraine was used. A pneumatic gun was used for launching (Fig. 3). This gun consists of a bore with a length of 4,000 mm and a diameter of 125 mm; a tank filled with compressed air; and a special membrane for rapid air intake [5].

A system for measuring the velocity of the object being launched is installed at the muzzle of the bore (Fig. 4). Velocity measurements were performed by registering the time between the rupture of two wires of diameter 0.3 mm, spaced at a distance of 100 mm from each other. Time measurement was performed using an E 20–10 analog-to-digital converter.

Fig. 3. Pneumatic gun.

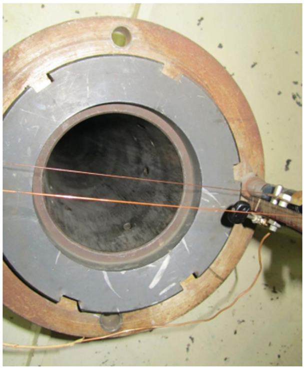

Fig. 4. Speed measurement system.

A 600 g indenter with a cylinder nose of diameter 20 mm was used for launching. Its main part is made of polyfoam, which together with two disks plays the role of a wad. The impactor part of the indenter is made of aluminum; and its weight, radius, and shape can be chosen depending on the objectives of the study.

Fig. 5 shows the scheme of an experimental setup (1 − gun bore, 2 – indenter, 3 – composite plate, 4 − crusher, 5 – support frame, 6 − clamping bar, 7 − rubber gaskets, 8 − base plate). A plasticine crusher was installed between the test sample and the base plate (Fig. 6). The difference between the length of the crusher before and after the test gives the maximum value of deflection.

The target for testing was fixed in a special box (Fig. 6), perpendicular to the direction of indenter movement (Fig. 6). The impact was delivered to the center of the outer surface of the composite plate.

Fig. 5. Scheme of an experimental setup.

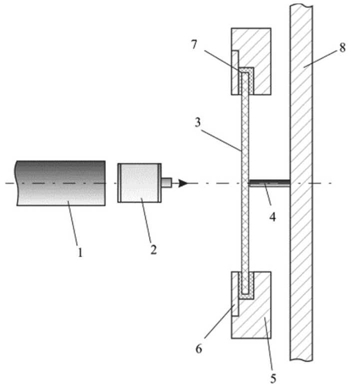

Fig. 6. A target and a box for tests.

In the experiment, the maximum deflection of a simple-supported flat fiberglass sample with a plan size of 500 × 500 mm was investigated. The fiberglass plastic consists of 11 layers: one front layer, two layers of glass-fiber mat, 8 layers of glass-fiber fabrics. For impregnation, the Crystic 1355 PA resin is used. The orientation of the layers is longitudinal-transverse. Layer thicknesses are $h_1 = h_2 = h_i = 0.6$ mm ($i = \overline{4, 11}$) and $h_3 = 0.9$ mm. Physical and mechanical characteristics of the front layer are $E_1^1 = E_2^1 = E_3^1 = 6.4$ GPa, $\nu_{12}^1 = \nu_{13}^1 = \nu_{23}^1 = 0.44$, $G_{12}^1 = G_{13}^1 = G_{23}^1 = 2.286$ GPa, $\rho^1 = 1600$ g/cm^3; of the glass-fiber mats, $E_1^2 = E_2^2 = E_3^2 = 6.4$ GPa, $\nu_{12}^2 = \nu_{13}^2 = \nu_{23}^2 = 0.44$, $G_{12}^2 = G_{13}^2 = G_{23}^2 = 2.286$ GPa, $\rho^2 = 100$ g/cm^3, $E_1^3 = E_2^3 = 15$ GPa, $E_3^3 = 7.689$ GPa, $\nu_{12}^3 = 0.12$, $\nu_{13}^3 = \nu_{23}^3 = 0.41$, $G_{12}^3 = 2.554$ GPa, $G_{13}^3 = G_{23}^3 = 2.184$ GPa, $\rho^3 = 450$ g/cm^3; of the glass-fiber fabrics, $E_1^i = E_2^i = 12.1$ GPa, $E_3^i = 6.331$ GPa, $\nu_{12}^i = 0.13$, $\nu_{13}^i = \nu_{23}^i = 0.41$, $G_{12}^i = 2.176$ GPa, $G_{13}^i = G_{23}^i = 2.06$ GPa, $\rho^i = 1580$ g/cm^3.

Deflections of three composite samples upon impact with velocities of 106, 97 and 84 m/s are studied. Table 1 shows the results of numerical and experimental studies of the maximum deflections of the fiberglass plastic in the middle of its rear side under the point of impact. The calculation assumes that the area of interaction between the indenter and the plate is a circle of the same diameter as the diameter of the impact cylinder. It can be seen that the results of calculations are in good agreement with the experimental data, which confirms the efficiency of the proposed method.

Table 1. Maximum deflections of composite at impact.

Velocity, m/s	Deflection, mm	
	Theory	Experiment
84	83.1	80
97	97.2	102
106	104.9	107

4 Conclusions

A theoretical and experimental approach to modeling the response of a laminated composite to the impact is proposed. To theoretically model the behavior of a composite, equations of the generalized model of layered structures are used, which allows taking into account the spatial nature of the deformation near the point of impact.

The possibilities of the proposed theoretical approach are shown using examples of a number of problems of calculating the stress-strained state of laminated composites with different sets of properties of layers. The probability of the obtained results is illustrated by their comparison with the calculation data of other authors, obtained using different two-dimensional theories.

An experimental study of the response of an eleven-layer glass-fiber composite to the impact by indenter was performed. For launching, the 600 g indenter was used. A pneumatic gun was used for launching the indenter, and crushers were used to register the maximum deflections. It is established that the calculation results and the experimental data are in good agreement.

The theory proposed has a wide field of application and allows for a valid description of the impact response of layered structures having a practically any composition of layers and pack thickness.

Acknowledgment. The work was supported in part by the budget program of the NAS of Ukraine KPKVK 6541230 “Supporting the development of priority areas of scientific research”.

References

1. Nicolais, L., Meo, M., Milella, E. (eds.): Composite Materials: A Vision for the Future. Springer, London (2011)
2. Slyvynskyi, V.I., Sanin, A.F., Kharchenko, M.E., Kondratyev, A.V.: Thermally and dimensionally stable structures of carbon-carbon laminated composites for space applications. In: Proceedings of the International Astronautical Congress (IAC), vol. 8, pp. 5739–5751 (2014)

3. EASA/AV CS-25. Certification specifications and acceptable means of compliance for large aeroplanes. European Aviation Safety Agency (2020)
4. Shupikov, A.N., Ugrimov, S.V., Smetankina, N.V., et al.: Bird dummy for investigating the bird-strike resistance of aircraft components. J. Aircr. **50**(3), 817–826 (2013). https://doi.org/10.2514/1.C032008
5. Smetankina, N., Kravchenko, O., Ugrimov, S., Yareshchenko, V.: Calculation and experimental study of the stress-strained state of laminated composites at impact loading. Paper presented at the 7th International Conference Space Technologies: Present and Future, Yuzhnoe State Design Office, Dnipro, 21–24 May 2019
6. Tiberkak, R., Bachene, M., Rechak, S., Necib, B.: Damage prediction in composite plates subjected to low velocity impact. Compos. Struct. **83**(1), 73–82 (2008). https://doi.org/10.1016/j.compstruct.2007.03.007
7. Richardson, M.O.W., Wisheart, M.J.: Review of low-velocity impact properties of composite materials. Compos. Part A Appl. Sci. Manuf. **27**(12), 1123–1131 (1996). https://doi.org/10.1016/1359-835X(96)00074-7
8. Abrate, S.: Impact on laminated composite materials. Appl. Mech. Rev. **44**(4), 155–190 (1991). https://doi.org/10.1115/1.3119500
9. Reddy, J.N.: On the generalization of displacement-based laminate theories. Appl. Mech. Rev. **42**(11S), S213–S222 (1989). https://doi.org/10.1115/1.3152393
10. Ugrimov, S.V., Shupikov, A.N.: Layered orthotropic plates. Generalized theory. Compos. Struct. **129**, 224–235 (2015). https://doi.org/10.1016/j.compstruct.2015.04.004
11 Kurennov, S.S.: Longitudinal-flexural vibrations of a three-layer rod. An improved model. J. Math. Sci. **215**(2), 159–169 (2016). https://doi.org/10.1007/s10958-016-2829-7
12. Nosier, A., Kapania, R.K., Reddy, J.N.: Low-velocity impact of laminated composites using a layerwise theory. Comput. Mech. **13**(5), 360–379 (1994). https://doi.org/10.1007/BF00512589
13. Pierson, M.O., Vaziri, R.: Analytical solution for low-velocity impact response of composite plates. AIAA J. **34**(8), 1633–1640 (1996). https://doi.org/10.2514/3.13282
14 Shupikov, A.N., Smetankina, N.V.: Non-stationary vibration of multilayer plates of an uncanonical form. The elastic immersion method. Int. J. Solids Struct. **38**(14), 2271–2290 (2001). https://doi.org/10.1016/S0020-7683(00)00166-9
15. Suvorova, I.G., Kravchenko, O.V., Baranov, I.A.: Mathematical and computer modeling of axisymmetric flows of an incompressible viscous fluid by the method of R-functions. J. Math. Sci. **184**(2), 165–180 (2012). https://doi.org/10.1007/s10958-012-0861-9
16. Sun, C.T., Chen, J.K.: On the impact of initially stressed composite laminates. J. Compos. Mater. **19**(6), 490–504 (1985). https://doi.org/10.1177/002199838501900601
17. Tan, T.M., Sun, C.T.: Wave propagation in graphite/epoxy laminates due to impact. Report No. NASA CR-168057. Purdue University, West Lafayette (1982)
18. Choi, I.H., Hong, C.S.: Low-velocity impact response of composite laminates considering higher-order shear deformation and large deflection. Mech. Compos. Mater. Struct. **1**(2), 157–170 (1994). https://doi.org/10.1080/10759419408945825
19. Patil, S., Reddy, D.M., Reddy, M.: Low velocity impact analysis on composite structures – a review. AIP Conf. Proc. **1943**(1), 020009 (2018). https://doi.org/10.1063/1.5029585
20. Cantwell, W.J., Morton, J.: The impact resistance of composite materials – a review. Composites **22**(5), 347–362 (1991). https://doi.org/10.1016/0010-4361(91)90549-V
21. Panettieri, E., Fanteria, D., Montemurro, M., Froustey, C.: Low-velocity impact tests on carbon/epoxy composite laminates: a benchmark study. Compos. Part B Eng. **107**, 9–21 (2016). https://doi.org/10.1016/j.compositesb.2016.09.057
22. Grigoliuk, E.I., Chulkov, P.P.: The theory viscoelasticity of multilayer shells with rigid filler at finite deflections. J. Appl. Mech. Tech. Phys. **5**, 109–117 (1964). (in Russian)

23. Smetankina, N.V., Shupikov, A.N., Sotrikhin, S.Y., Yareshchenko, V.G.: A noncanonically shape laminated plate subjected to impact loading: theory and experiment. J. Appl. Mech. **75** (5), 051004 (2008). https://doi.org/10.1115/1.2936925
24 Shupikov, A.N., Ugrimov, S.V., Kolodiazhny, A.V., Yareschenko, V.G.: High-order theory of multilayer plates. The impact problem. Int. J. Solids Struct. **35**(25), 3391–3403 (1998). https://doi.org/10.1016/S0020-7683(98)00020-1
25. Dinnik, A.N.: Selected Works. Academy of Sciences of the Ukrainian SSR Publ, Kiev (1952).(in Russian)
26. Zukas, J.A., Nicholas, T., Swift, H.F., et al.: Impact Dynamics. Wiley, New York (1982)