Multi-objective Optimization of Co-cured Composite Laminates with Embedded Viscoelastic Damping Layer

Lijian Pan, Boming Zhang and Fuhong Dai[†]

Centre for Composite Materials and Structure, Harbin Institute of Technology, Harbin 150080, China [Manuscript received May 20, 2008, in revised form October 7, 2008]

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Presented herein is a methodology for the multi-objective optimization of damping and bending stiffness of cocoured composite laminates with embedded viscoelastic damping layer. The embedded viscoelastic damping layer is perforated with a series of small holes, and the ratio of the perforation area to the total damping area is the design variable of the methodology. The multi-objective optimization is converted into a single-objective problem by an evaluation function which is a liner weigh sum of the two sub-objective functions. The proposed methodology was carried out to determine the optimal perforation area ratios of two viscoelstic layers with different perforation distance embedded in two composite plates. Both the optimal perforation area ratios are approximate to 2.2%. However, the objective value of the plate with greater perforation distance in embedded viscoelatic layer is much greater.

KEY WORDS: Composite laminates; Viscoelastic damping layer; Co-cured; Multi-objective optimization

1. Introduction

In recent years, co-curing damping materials in composites has been shown to be successful in greatly increasing the damping of composite structures^[1-7]. Co-curing refers to the process of inserting viscoelastic materials within composite laminates before the composite is cured, therefore the embedded viscoelastic materials should undergo the temperature and pressure cycle which is necessary to cure the composite.

Although co-curing damping materials in composites does not decrease the tensile and compression stiffness or strength, the presence of the damping layer decreases the bending stiffness of composite structure remarkably^[8]. It cannot meet the requirement of integrated design of structure. A common technology to solve this problem is perforating with a series of small holes in the embedded viscoelastic damping layer as shown in Fig. 1. During the co-curing, the resin flows through the damping layer and completely couple the structure that will enhance the bending stiffness remarkably. Although the perforating with holes in the viscoelastic damping layer can enhance the bending stiffness, it will result in the loss of damping. The perforation area should be designed neither too big nor too small. Thus, the optimization of perforation area is a crucial step for coordination of the contradiction between the damping and bending stiffness of co-cured composite laminates with viscoelastic damping layer.

In this article, a methodology for the multiobjective optimization of damping and bending stiffness of co-cured composite laminates is presented. The optimization is accomplished by converting the multi-objective problem into a single-objective problem by construction an evaluation function. The evaluation function is a liner weigh sum of the two subobjective functions (loss factor and bending stiffness). The two sub-objective functions are constructed by

E-mail address: daifh@hit.edu.cn (F.H. Dai).

fitting the curves of the loss factor and bending stiffness vs. perforation area ratio respectively, and the weigh numbers are determined by the margin of each sub-objective function.

2. Mathematical Model

2.1 Evaluation function

Optimization requires a suitable objective function which is a function of user specified design variables^[9,10]. The objective function is then minimized or maximized by the optimization algorithm.

For a multi-objective optimization, we can construct a new function called evaluation function to convert a multi-objective problem into a singleobjective problem for simplification. There are many ways to construct an evaluation function. One of the most important methods is liner weighted sum method, and the evaluation function is given by:

$$f(X) = \sum_{i=1}^{L} \omega_i f_i(X) \tag{1}$$

where, f(X) is the new objective function; X is the design variables; Ω_i is weigh number for each subobjective function $f_i(X)$; L is the amount of subobjective. Each weigh number ω_i should be normalized and non-negative written below:

$$\sum_{i=1}^{L} \omega_i = 1, \ \omega_i \ge 0 (i = 1, 2, \cdots, L)$$
(2)

To be mentioned, the magnitude of weigh number represents importance of each sub-objective, and the more important the sub-objective, the greater the weigh number ω_i . Commonly, there is significant difference in the value of each sub-objective due to the different dimension. This will result in a hard reflection of the importance of each sub-objective. Therefore, the each sub-objective function should be

 $[\]dagger$ Corresponding author. Prof.; Tel.: +86 451 86414145;



Fig. 1 Schematic description of co-cured composite laminates with embedded perforated viscoelastic damping layer

dimensionless. The most common technique is given by: $\Gamma(U)$

$$f_i(X) = \frac{F_i(X)}{\min_{x \in \Omega} F_i(X)}$$
(3)

where, $F_i(X)$ is the *i*th sub-objective function with dimension, Ω is the feasible zone.

The present optimization study aims at finding an optimal perforation area for a greater damping with less loss in bending stiffness of co-cured composite structure. Thus, there are two sub-objective in the present study, they are loss factor and bending stiffness. So we can rewrite the Eq. (1) as follows:

$$f(X) = \omega_1 f_1(X) + \omega_2 f_2(X) \tag{4}$$

$$X = \frac{A_{\rm p}}{A_{\rm t}} \tag{5}$$

where, ω_1 and $f_1(X)$ are weigh number and subobjective function of loss factor respectively; ω_2 and $f_2(x)$ are weigh number and sub-objective function of bending stiffness respectively; the design variables; Xis the ratio of the perforation area to the total area; A_p is the perforation area; A_t is the total area.

2.2 Sub-objective function

In this section, the two sub-objective functions will be built. Robinson and Kosmatka^[11] have investigated the damping and bending stiffness of co-cured composite laminates with different perforation in the embedded viscoelatic layer. Results show that the damping and stiffness can be very sensitive to perforation spacing and size. There is very little change in the stiffness of the plates for a damping area less than 95%. However for damping areas from 95% to 100%can be seen a sharp decrease in the stiffness of the plates. There is very little change in the damping of the first bending mode for a damping area less than 95%. However for damping areas from 95% to 100% a sharp increase in the first mode damping can be seen. It is not difficult to find that the curves of loss factor and bending stiffness vs. perforation area ratio can be fitted well by power function:

$$\beta(X) = a_1 X^{b_1} \tag{6}$$

$$B(X) = a_2 X^{b_2} \tag{7}$$

where, $\beta(X)$ is loss factor; B(X) is bending stiffness; a_1, b_1, a_2, b_2 are coefficients to be determined by curve fitting method. The two sub-objective functions should be dimensionless, and it can be performed by Eq. (3):

$$f_1(X) = \frac{\beta(X)}{\min\beta(X)} = \frac{\beta(X)}{\beta_1} \tag{8}$$

$$f_2(X) = \frac{\beta(X)}{\min B(X)} = \frac{B(X)}{B_0} \tag{9}$$

$$\beta_1 = \beta \ (X = 1), \ B_0 = B \ (X = 0)$$
 (10)

Obviously, when X equals to 1, there is no viscoelastic damping layer. So, β_1 indicates the loss factor of undamped composite structure. When X equals to 0, there is no perforation in viscoelastic damping layer. So B_0 indicates the bending stiffness of fully damped composite structure. The value of the first sub-objective can be explained as the reinforced amplitude of damping due to the embedding viscoelastic damping layer, and the second sub-objective can be explained as the reinforced amplitude of bending stiffness due to the perforation in the embedded viscoelastic damping layer. Thus, the greater the subobjective values, the better the design sachem. To increase the fist sub-objective value, we must decrease the perforation area, however, to increase the second sub-objective value, we must increase the perforation area. Thus, the optimization of perforation area is urgently needed.

By substituting Eqs. (6) and (7) into Eqs. (8) and (9) respectively, the two sub-objective can be rewritten as follows:

$$f_1(X) = \frac{a_1 X^{b_1}}{\beta_1} \tag{11}$$

$$f_2(X) = \frac{a_2 X^{b_2}}{B_0} \tag{12}$$

2.3 Weigh number

The weigh numbers for each sub-objective will be determined. We all know that the smaller the perforation area, the greater the loss factor of co-cured composite structure. And there is a minimum loss factor when there is no viscoelastic damping layer and the perforation area ratio equals to 1. Just the reverse, the greater the perforation area is, the greater the bending stiffness of co-cured composite structure is. And there is a minimum bending stiffness when there is no perforation in viscoelastic damping layer and the perforation area ratio equals to 0. Accordingly, the span of each sub-objective function can be written as follows:

$$1 \neq f_1(X) \le \frac{\beta_0}{\beta_1} \tag{13}$$

$$1 \neq f_2(X) \le \frac{B_1}{B_0}$$
 (14)

According to the definition of margins, the margins for each sub-objective function can be calculated:

$$\Delta f_1(X) = \frac{\beta_0 / \beta_1 - 1}{2}$$
(15)

$$\Delta f_2(X) = \frac{B_1/B_2 - 1}{2} \tag{16}$$

 Table 1
 The dimension of perforation in embedded viscoelastic layer

Plate	Undampe d	Fully damped	1	2
S/\rm{mm}	-	-	25	50
D/mm	—	—	2, 3, 4, 5, 6, 8, 12, 16, 24	4, 6, 8, 10, 12, 16, 24, 32, 48
X	1	0	0.00502, 0.0113, 0.0201, 0.0314	, 0.0452, 0.0804, 0.181, 0.322, 0.723

The weigh number for each sub-objective function should meet the normalized and non-negative condition in Eq. (2) and can be determined as follows:

$$\omega_1 = \frac{\Delta f_2(X)}{\Delta f_1(X) + \Delta f_2(X)} = \frac{B_1/B_2 - 1}{\beta_0/\beta_1 + B_1/B_2 - 2}$$
(17)
$$\omega_2 = \frac{\Delta f_1(X)}{\Delta f_1(X) + \Delta f_2(X)} = \frac{\beta_0/\beta_1 - 1}{\beta_0/\beta_1 + B_1B_2 - 2}$$
(18)

The weigh numbers in Eqs. (17) and (18) can keep each sub-objective in balance in order of magnitude. When the value span of the sub-objective function is greater and so as the margin, however, the weigh number will be decreased.

2.4 Optimization study

Since both the greater the sub-objective value, the better the design sachem is. By substituting subobjective functions Eqs. (11), (12) and weigh numbers Eqs. (17), (18) into Eq. (4), then the multioptimization problem is reduced to the following maximization problem

$$f(X) = \frac{1}{\beta_0/\beta_1 + B_1 B_2 - 2} \left[\frac{(B_1/B_2 - 1)\alpha_1 X^{\mathbf{b}_1}}{\beta_1} + \frac{(\beta_0/\beta_1 - 1)\alpha_2 X^{\mathbf{B}_2}}{B_0} \right]$$
(19)

The optimization problem solved is expressed as maximization of f(X). The better of the design scheme is, the bigger of the value of the evaluation function is. To get the maximum value of the evaluation function f(X), we can derivate the evaluation function and order the derivation of the evaluation function to be zero:

$$f'(X) = 0 \tag{20}$$

That is:

$$\frac{1}{\beta_0/\beta_1 + B_1/B_2 - 2} \left[\frac{(B_1/B_2 - 1)a_1b_1X^{b_1 - 1}}{\beta_1} + \frac{(\beta_0/\beta_1 - 1)a_2b_2X^{b_2 - 1}}{B_0} \right] = 0$$
(21)

By solving the Eq. (21), then:

$$X^{b_1-b_2} = -\frac{(\beta_0/\beta_1 - 1)a_2b_2\beta_1}{(B_1/B_2 - 1)a_1B_1B_0}$$
(22)

By substituting Eqs. (17) and (18) into the Eqs. (22), the optimal cab be rewritten as follows:

$$X = (-\omega_2 a_2 b_2 \beta_1 / \omega_1 a_1 b_1 B_0)^{\frac{1}{(b_1 - b_2)}}$$
(23)

When the value of the ratio of the perforation area to the total area meet the Eq. (23), we can get the maximum of the evaluation function f(X). Then, the value



Fig. 2 Schematic description of embedded viscoelastic damping layer with a series of perforation

of X in Eq. (23) is an optimal ratio of the perforation area to the total damping area.

3. Results and Discussion

In this section, two co-cured composite laminates were optimized by the proposed model. Both the dimension of the composite laminates is $200 \text{ mm} \times 100 \text{ mm}$ (length×width) and the embed viscoelastic damping layer perforated a series of small hole as shown in Fig. 2. The dimensions of the perforation are listed in Table 1. The loss factor for laminated composite structures with integral damping layers can be calculated by modal strain energy (MSE) method^[12–14], and the *n*th modal loss factor is given by:

$$\beta_n = \left(\frac{D_{\rm v} + D_{\rm r} + D_{\rm c}}{U_{\rm t}}\right)_n \tag{24}$$

$$(D_{\mathbf{v}})_n = \left(\sum_{i=1}^k \beta_{\mathbf{v}} U_{\mathbf{v}}^i\right)_n \tag{25}$$

$$(D_{\mathbf{r}})_n = \left(\sum_{i=1}^l \beta_{\mathbf{r}} U_{\mathbf{r}}^i\right)_n \tag{26}$$

$$(D_{\rm c})_n = \left(\sum_{i=1}^m \beta_{\rm c} U_{\rm c}^i\right)_n \tag{27}$$

where, β_n is the *n*th modal loss factor of the structure, D_v is the energy dissipation in the viscoelastic materials, D_r is the energy dissipation in the resin in perforation, D_c is the energy dissipation due to the fiber-reinforced composites, U_t is the total strain energy in the structure, β_v is the loss factor of the viscoelastic damping material, U_v^i is the strain energy of the *i*th element for the damping layer, β_r is the loss factor of the in the resin in perforation, U_r^i is the strain energy of the *i*th element for the resin in perforation, β_c is the loss factor of fiber-reinforced composites, U_c^i is the strain energy of the *i*th element for fiber reinforced composites, k, l and m are respectively the total elements in the damping layers, the resin in perforation and fiber-reinforced laminates.

Table 2The loss factor and first bending mode frequency of undaped and fully damped structures and weigh numbers Undamped Fully damped Weigh numbers

Undamped		Fully damped		Weigh numbers		
	β_1	Frequency/Hz	β_1	Frequency/Hz	ω	ω_2
	0.032	189.4	0.2469	295.2	0.87386	0.02754



Fig. 3 Loss factor vs. perforation area ratio of plate 1



Fig. 4 First bending mode frequency vs. perforation area ratio of plate 1

The carbon fiber reinforced plastic properties considered here are given as follows: $E_{11}=120$ GPa, $E_{22}=9$ GPa, $G_{12}=4.3$ GPa, $v_{12}=0.29$, $\rho=1635$ kg/m³, $\beta_c=0.0132$. The properties of resin flowed in perforation are given as: E=3.2 GPa, $\rho=1250$ kg/m³, $\beta_r=0.06$. These factors were provided by Harbin FRP research institute FRP factory. The properties of viscoelastic material are given as: E=7.8 MPa, $\rho_v=1350$ kg/m³, Poisson's ratios v=0.48, $\beta_v=0.6$.

The damping and bending stiffness properties of undamped and fully damped composite plates for first bending mode are listed in Table 2. Then, the weigh number for each sub-objective can be calculated by Eqs. (17) and (18), also listed in Table 2.

The modal loss factors and mode frequencies vs the perforation area ratio for the first bending mode of vibration are respectively plotted in Figs. 3–6, and so as the corresponding fitting curves. It is observed that relationship between the loss factor, bending stiffness and perforation area ratio is similar to the experimental result in literature [11]. And the fitting curves agree well with the curves by calculation with a confidence of 0.95. The parameters of fitting curves are listed in Table 3.

The optimal perforation area ratio X can be calculated by substituting the weigh number of each sub-



Fig. 5 Loss factor vs. perforation area ratio of plate 2



Fig. 6 First bending mode frequency vs. perforation area ratio of plate 2

objective (listed in Table 2) and the coefficients a_1 , b_1 , a_2 , b_2 (listed in Table 3) into the Eq. (23). The optimal perforation area ratios calculated are listed in Table 4. It is observed that the optimal perforation area ratios of two plates are approximate to 2.2% which is less than 5% suggested by literature [11].

Table 3 The fitting result of the curves of loss factor and bending stiffness vs. perforationratio plate loss factor Bending stiffness

Plate	Loss factor		Bending stiffness		
1 late	a_1	b_1	a_2	b_2	
1	0.00785	-0.52177	299.4159	0.05598	
2	0.02457	-0.36173	293.46203	0.07109	

Finally, we can get the objective value, damping and bending stiffness properties by substituting the optimal perforation area ratio into the evaluation function Eq. (19) and each sub-objective function Eqs. (11) and (12), respectively. For the plate 1, the optimum perforation area ratio is 0.759%, the damping increased by 3.3 times while the bending stiffness decreased by 20.5%. For plate 2, the optimum perforation area ratio is 3.43%, the damping increased by 3.5 times while the bending stiffness decreased by 22.9%.

 Table 4
 The optimal perforation area ratio and relative optimization results plate optimal area ratio

Plate	Optimal area ratio	Objective value	Loss factor	First bending
	X	f(X)	β	natural frequency/Hz
1	0.0221	3.834	0.565	242.98
2	0.0222	6.479	0.1076	222.88

4. Conclusions

A methodology for the multi-objective optimization of co-cured composite laminates with embedded viscoelastic damping layer has been presented. The optimization is performed by an evaluation function which is the liner weigh sum of the two sub-objective functions (loss factor and bending stiffness). The curves of loss factor and bending stiffness vs. perforation area ratio of embedded viscoelastic damping layer can be well fitted respectively by power function with a confidence of 0.95. The each sub-objective function is dimensionless and their weigh number is determined by their margins to keep the balance of each sub-objective.

We studied an optimization for two co-cured composite plates with different perforation distance in embedded viscoelatic layer. The results show that both the optimal perforation area ratios are less than 5% suggested by literature 11, the method put forward is validated.

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